Overcoming Non-Discernibility through Mode-Sequence Analytic Redundancy Relations in Hybrid Diagnosis and Estimation

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ABSTRACT

Estimating the mode of operation of a hybrid system means to account for its interleaved continuous and discrete dynamics from a partial set of observations. The quality of the estimation obviously depends on the level of diagnosability of the system, which has been shown to depend on the discrete and the continuous observations as a whole. The case of two modes showing very similar, if not the same, continuous dynamic input/output behavior is critical when the discrete events do not allow discrimination. In this case, these modes, so called non-discernible modes, are accepted to be not discriminable. This paper uses the concept of *Mode-Sequence Analytic Redundancy Relation* which is shown to achieve discriminability of such non-discernible modes in many cases. A focusing test based on mode-sequence ARRs is then proposed to improve hybrid state estimation.

1 INTRODUCTION

Present-day artifacts, aircrafts, mobile robotic devices, space probes, or production plants, benefit from multi-domain technologies in interaction, which allow the engineers to precisely design them at the nearest of every situation that may be encountered. They hence exhibit complex patterns of behavior and numerous nominal modes of operation in order to satisfy the high demand on performance and dependability. A precise knowledge of the current mode of operation/failure and the current state of the physical entities that capture the continuous evolution of the physical system are important prerequisites for this supervision/control task. This is even more true when the system may suffer disturbances and faults. In the latter case, knowing the health status of the different system components, and consequently its mode of operation, is essential for proceeding to the proper reconfiguration actions or adapt appropriately the control laws. It is the task of the state estimator to infer the mode and the state from the partial observations that are available and a mathematical model of the physical system.

An active track of research of the last decade is based on hybrid modeling paradigms that integrates both, the continuously-valued state evolution and the interleaved discrete mode changes in a comprehensive manner. The majority of research in hybrid estimation and diagnosis deals with the computational complexity of the estimation task. An exact method would require to consider every possible mode sequence with its associated continuous evolution, resulting in an inevitable blowup of the number of state estimates, also called hypotheses. This is why numerous sub-optimal, but computationally feasible, estimation methods have been proposed (Ackerson and Fu, 1970; Blom and Bar-Shalom, 1988; de Freitas, 2002; Hofbaur and Williams, 2004; Narasimhan and Biswas, 2002; Benazera *et al.*, 2002; Benazera and Travé-Massuyès, 2009; Verma *et al.*, 2004).

In this manuscript, however, we focus onto the root cause of ambiguity in hybrid estimation. The quality of the estimation obviously depends on the level of diagnosability of the system, which has been shown to depend on the discrete and the continuous observations as a whole (Bayoudh *et al.*, 2008a). The case of two modes showing very similar, if not the same, continuous dynamic input/output behavior is critical when the discrete events do not allow discrimination. In this case, these modes, so called non-discernible modes (Cocquempot *et al.*, 2004), are not discriminable and the cause of mode estimation ambiguity.

Whereas state of the art methods fail in discriminating among non-discernible modes, we propose to use the concept of *Mode-Sequence Analytic Redundancy Relation* which is shown to achieve discriminability. Following the ideas introduced in (Rienmüller *et al.*, 2009), mode-sequence ARRs are then shown to be particularly suited for focusing the set of mode hypotheses to be considered by a hybrid (full) state estimator.

The paper is organized as follows. Section 2 presents the hybrid formalism. Section 3 considers the hybrid estimation problem, rises the issue of non-discernible modes, demonstrated with a simple 2-mode system that shows this effect, and introduces mode-sequence ARRs as a solution to discriminate these modes. An algorithm integrating a mode-sequence ARR test with a multi-mode filtering process is then presented. Finally, section 4 concludes the paper and provides some open issues for further investigation.

2 HYBRID MODEL

The term hybrid system or hybrid model stands for a large variety of different mathematical models that combine continuously-valued and discretely-valued dynamics. Depending on the application area, one puts emphasis on either of the dynamics and thus provides quite divergent models for such systems.

In order to focus on the relevant aspects for our problem of estimation and diagnosis for non-discernible modes, we use a rather simple hybrid model that is related to so-called jump-linear/switched-linear hybrid systems (Vidal *et al.*, 2003). The continuously-valued dynamics of the model is described through the linear time-discrete state-space model, with sampling period T_s, of the form

$$\mathbf{x}_k = \mathbf{A}_i\mathbf{x}_{k-1} + \mathbf{B}_i\mathbf{u}_{k-1} \quad (1)$$
$$\mathbf{y}_k = \mathbf{C}_i\mathbf{x}_k + \mathbf{D}_i\mathbf{u}_k, \quad (2)$$

where $\mathbf{x}_k \in \Re^{n_x}, \mathbf{u}_k \in \Re^{n_u}$ and $\mathbf{y}_k \in \Re^{n_y}$ denote the valuations of the (continuously-valued) state, input, and output at time $t = kT_s$. The index i specifies the dependency of the dynamics on the mode or discrete state $\mathbf{x}_{d,k} = q_i$ of the model. As a consequence, the valuation of the mode at the time $t = kT_s$, $\mathbf{x}_{d,k}$, defines the continuous evolution of the state $\mathbf{x}$ in the time-interval $(k-1)T_s < t \leq kT_s$. In the following, we will abbreviate the multimode continuous part of the model with $\Xi := \langle \zeta, Q, C \rangle$, where $\zeta = \{\mathbf{x}, \mathbf{u}, \mathbf{y}\}$ denotes all continuously-valued (but time-discrete) variables, $Q = \{q_1, \ldots, q_l\}$ denotes the set of all modes and C stands for the mode-dependent state-space model (1-2).

The hybrid systems/hybrid model framework assumes an infrequent, but abrupt discrete evolution of the model. In other words, one can assume that the system exhibits only a single mode change within one sampling period. Our discrete-time model further assumes that the abrupt mode change occurs immediately after the sampling time-point $t = kT_s$ so that the new mode defines the continuous evolution within the following sampling period. Figure 1 visualizes the temporal ordering of the discrete and continuously-valued dynamics. Discrete mode changes are modeled through a discrete event model (DES). In detail, we write

$$M := \langle Q, \Sigma, T, Q_0 \rangle, \quad (3)$$

for the DES model part with the set of modes $Q = \{q_1, \ldots, q_l\}$, the set of events $\Sigma = \{\sigma_1, \ldots, \sigma_{n_e}\}$, the transition function $T : Q \times \Sigma \to Q$ and the set of initial states Q_0. Of course, we do assume that not all events σ_j can be observed. As a consequence, we define the sub-set $\Sigma_O \subseteq \Sigma$ of observable events and use the variable e to denote the event observation. The valuation of e at time-step k can be either an observable event $\sigma_j \in \Sigma_O$ or the empty valuation ϵ whenever there was no event or an unobservable event within the last sampling period, i.e. $e_k \in \{\Sigma_O, \epsilon\}$.

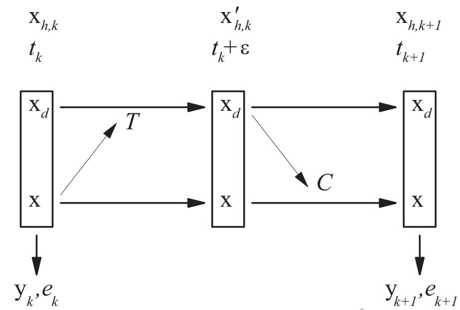

Figure 1: Dynamic evolution of the hybrid model.

Figure 2: Discrete model with two modes q_1 and q_2.

Summing up we denote the hybrid model through the pair

$$S = \langle \Xi, M \rangle \quad (4)$$

with the hybrid state

$$\mathbf{x}_h = \langle \mathbf{x}_d, \mathbf{x} \rangle \quad (5)$$

that is comprised of the mode $\mathbf{x}_d$ and the continuously-valued state $\mathbf{x}$.

2.1 Example

For the following analysis we introduce a hybrid model with two modes $Q = \{q_1, q_2\}$. The mode transitions $q_1 \to q_2$ and $q_2 \to q_1$ are due to a common observable event $\Sigma = \Sigma_O = \{\sigma\}$, as shown in figure 2. Furthermore, we specify an ambiguous initial mode of the DES through $Q_0 = \{q_1, q_2\}$, i.e. both modes of the system are potential initial states. This fact, as well as the unspecific transition observation through the common event σ prevents the mode estimation through observing the event sequence $\{e_1, \ldots, e_{k-1}\}$ only.

The associated continuous dynamics Ξ are of form (1-2) with the following system parameters

$$\mathbf{A}_1 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.5 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} 0 & 1 \\ -0.4 & 1.3 \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix},$$
$$\mathbf{B}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \mathbf{C}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}^T, \mathbf{C}_2 = \begin{bmatrix} -2.1 \\ 3 \end{bmatrix}^T, \mathbf{D}_1 = \mathbf{D}_2 = 1. \quad (6)$$

These parameters define two dynamic models with identical transfer function

$$\mathbf{G}_1 = \mathbf{G}_2 = \frac{z^2 + 1.7z - 1.7}{z^2 - 1.3z + 0.4} = \frac{3z - 2.1}{z^2 - 1.3z + 0.4} + 1. \quad (7)$$

One could also say that the dynamic models for the two modes define two distinct state-space representations of a common dynamic input/output behavior. This is also the reason why hybrid observation is non-trivial for this system.

3 HYBRID OBSERVATION AND DIAGNOSIS

The hybrid estimation task can be formulated as follows:

> **Hybrid Estimation:** Compute on the basis of the hybrid model S and the discrete-time sequences for the inputs $\{\mathbf{u}_0, \mathbf{u}_1, \ldots \mathbf{u}_k\}$, measurements $\{\mathbf{y}_0, \mathbf{y}_1, \ldots \mathbf{y}_k\}$, and events $\{e_0, e_1, \ldots e_k\}$ the estimate $\hat{\mathbf{x}}_k$ for the hybrid state $\mathbf{x}_h$ at time-point $t = kTs$, comprised of the estimate $\hat{\mathbf{x}}_{d,k}$ for the mode $\mathbf{x}_d$ and the estimate $\hat{\mathbf{x}}_k$ for the continuously-valued state $\mathbf{x}$.

The interwoven mode-transitions and continuous dynamics lead to a set of possible continuously-valued evolutions of the system and it is not possible to determine estimates for the mode (or mode sequence) and the continuously-valued evolution separately. As a consequence, a hybrid estimator has to track *all possible mode-sequences* with their associated continuously-valued evolution and evaluate their compatibility with the known/observed system variables. This leads to an exponentially increasing number of hypotheses over time which is computationally unfeasible. Many estimation techniques deal with exactly this computational issue and provide sub-optimal hybrid estimators that limit the observation history and merge hypotheses (Ackerson and Fu, 1970; Blom and Bar-Shalom, 1988) or focus onto the sub-set of likely hypotheses only (Li and Bar-Shalom, 1996; de Freitas, 2002; Hofbaur and Williams, 2004; Verma *et al.*, 2004). The continuously-valued estimate is then computed algorithm-dependent with specifically instantiated dynamic filters.

Hybrid diagnosis, in contrast, only computes the estimate $\hat{\mathbf{x}}_{d,k}$ for the mode of operation/failure $\mathbf{x}_{d,k}$. However, because of the interwoven continuously-valued and discrete evolution of the system, one still has to evaluate both dynamics to provide the mode-estimate sought for. Similar to hybrid estimation, one has to evaluate the possible mode-hypotheses and their consistency with the known and observed information. The consistency test is formulated in terms of a so called residual $\mathbf{r}$ which provides $\mathbf{r}_k = \mathbf{0}$ for consistent hypotheses. The evaluation of the residual is typically done through residual filters (Patton and Chen, 1997), system identification (Ljung and Glad, 1994) or through so called *Analytic Redundancy Relations (ARR)* (Gertler, 1991; Cocquempot *et al.*, 2004; Bayoudh *et al.*, 2008b).

Analytic Redundancy Relations associate the inputs ($\mathbf{u}$) and outputs ($\mathbf{y}$) over a limited observation horizon of length $p+1$. The vectors

$$U_k \;:=\; [\mathbf{u}_{k-p}^T, \ldots, \mathbf{u}_k^T]^T,\;\; Y_k := [\mathbf{y}_{k-p}^T, \ldots, \mathbf{y}_k^T]^T \tag{8}$$

define the compound vectors of inputs and outputs over the observation horizon so that we can reformulate the system equations (1-2) for the mode q_i as the analytic expression

$$Y_k = \mathbf{O}_i \mathbf{x}_{k-p} + \mathbf{L}_i U_k \tag{9}$$

with the matrices

$$\mathbf{O}_i := \begin{bmatrix} \mathbf{C}_i \\ \mathbf{C}_i\mathbf{A}_i \\ \vdots \\ \mathbf{C}_i\mathbf{A}_i^p \end{bmatrix} \tag{10}$$

and

$$\mathbf{L}_i := \begin{bmatrix} \mathbf{D}_i & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{C}_i\mathbf{B}_i & \mathbf{D}_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{C}_i\mathbf{A}_i^{p-1}\mathbf{B}_i & \cdots & \mathbf{C}_i\mathbf{B}_i & \mathbf{D}_i \end{bmatrix}. \tag{11}$$

If one selects the observation horizon, or the parameter p, sufficiently large, one can always compute an orthogonal matrix $\mathbf{\Omega}_i$ for $\mathbf{O}_i$ so that

$$\mathbf{\Omega}_i \mathbf{O}_i = \mathbf{0}.$$

This fact allows us to eliminate the state ($\mathbf{x}_{k-p}$) dependency of (9) through (left) multiplication with $\mathbf{\Omega}_i$ and we obtain the ARR for mode q_i

$$\mathbf{\Omega}_i Y_k \;=\; \mathbf{\Omega}_i \mathbf{O}_i \mathbf{x}_{k-p} + \mathbf{\Omega}_i \mathbf{L}_i U_k \tag{12}$$

$$\mathbf{\Omega}_i Y_k \;=\; \mathbf{\Omega}_i \mathbf{L}_i U_k. \tag{13}$$

Based on this ARR we can define a residual-vector $\mathbf{r}_{i,k}$ for mode q_i

$$\mathbf{r}_{i,k} := \mathbf{\Omega}_i Y_k - \mathbf{\Omega}_i \mathbf{L}_i U_k. \tag{14}$$

so that the associated consistency test[1]

$$\tilde{r}_{ij,k} := \begin{cases} 0 & \text{if } |r_{ij,k}| = 0 \\ 1 & \text{else} \end{cases}, \;\; j = 1, \ldots, m_i \tag{15}$$

that provides an m_i-dimensional binary residual vector for mode q_i at the time-step k as

$$\tilde{\mathbf{r}}_{i,k} := [\tilde{r}_{i1,k}, \ldots, \tilde{r}_{im_i,k}]^T. \tag{16}$$

A parallel evaluation of (14) for all modes $q_i, i = 1, \ldots, l$ can be used for hybrid diagnosis (Cocquempot *et al.*, 2004). Furthermore, it is advantageous to use an additional discrete-event diagnoser (Sampath *et al.*, 1995) that utilizes the discrete dynamics model to focus the diagnosis result onto few modes (Bayoudh *et al.*, 2008a; 2008b). In (Rienmüller *et al.*, 2009) we were able to show that the ARR based consistency test also provides additional focusing capabilities for hybrid full state estimation.

3.1 Non-discernible Modes

The ARR test (14-15) evaluates the consistency of the mode hypothesis $\mathbf{x}_{d,k} = q_i$ with the input and output sequences. More specifically, because of the observation horizon that is defined through $p+1$ tests

$$\mathbf{x}_{d,j} = q_i, j = k-p, \ldots, k \Rightarrow \tilde{\mathbf{r}}_{i,k} \equiv \mathbf{0}. \tag{17}$$

However, it is possible that more than one mode in a system provides consistent residuals, i.e. residuals that

[1] Of course, any real world application has to take disturbances (measurement noise, etc.) into account, so that the condition has to be re-formulated with an upper limit ε_{ij} for $|r_{ij,k}|$.

evaluate to zero. The example above can illustrate this fact. The evaluation-form of the ARRs (14) for $p = 2$

$$\begin{aligned}\mathbf{r}_{1,k} &:= \mathbf{\Omega}_1 Y_k - \mathbf{\Omega}_1 \mathbf{L}_1 U_k \\ &= \left[\begin{array}{ccc} 0.2369 & -0.7701 & 0.5923 \end{array}\right] Y_k \\ &\quad - \left[\begin{array}{ccc} -1.0070 & 1.0070 & 0.5923 \end{array}\right] U_k \\ &= \mathbf{\Omega}_2 Y_k - \mathbf{\Omega}_2 \mathbf{L}_2 U_k =: \mathbf{r}_{2,k}\end{aligned} \tag{18}$$

is identical for both modes! Of course, this is no coincidence, but a direct consequence of the identical input/output behaviors of both modes, a fact that was already suggested by the two identical transfer functions (7). Modes of this type are called *non-discernible* in the literature (Cocquempot *et al.*, 2004; Bayoudh *et al.*, 2008b)). In detail, one can determine this property through the following test:

Proposition 1 *(Cocquempot* et al.*, 2004) Two modes q_i and q_j of a hybrid system are non-discernible iff the matrices $\mathbf{O}_i, \mathbf{O}_j, \mathbf{L}_i$ and $\mathbf{L}_j$ with $p = n_x$ satisfy*

$$Null\left(\mathbf{O}_i^T\right) = Null\left(\mathbf{O}_j^T\right) \subseteq Null\left(\Delta_{ij}^T\right) \tag{19}$$

with $\Delta_{ij} := \mathbf{L}_i - \mathbf{L}_j$.

These conditions can be tested efficiently through rank tests:

$$\text{Rank}(\mathbf{O}_i) = \text{Rank}(\mathbf{O}_j) = \text{Rank}\left([\mathbf{O}_i, \mathbf{O}_j, \Delta_{ij}]\right) \tag{20}$$

One can detect this observation defect or better the problematic modes of operation through a careful system's analysis at compile time. Evaluating the ARRs for these modes does not provide enough evidence to diagnose the specific mode of operation or failure. However, one can use the evaluation of the ARRs to identify (in almost all cases) mode transitions as it can be seen in Fig. 4 for the simulation experiment of Fig. 3. The evaluation to a non-zero residual value is due to the mode transition within the observation horizon

$$(k - p)T_s \ldots kT_s.$$

This is also the reason why ARR based hybrid diagnoser/observer can only provide the estimate for the mode with a delay of (at least) p time-steps (unless the observer uses also other information, such a discrete events etc.).

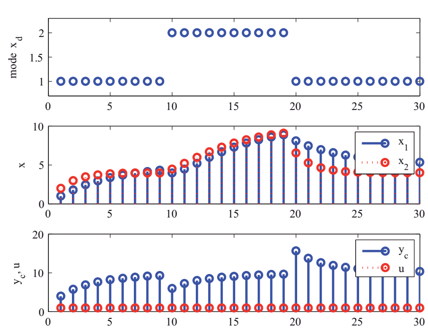

Figure 3: Simulation of the 2-mode example with 2 mode changes.

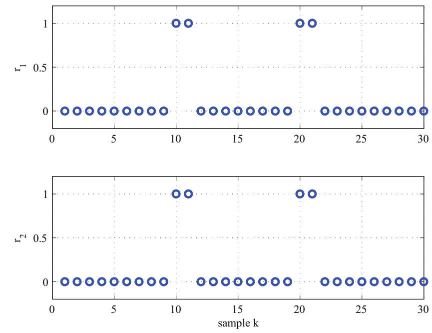

Figure 4: Residuals $\mathbf{r}_{1,k}$ and $\mathbf{r}_{2,k}$ for the simulation experiment.

3.2 Mode-sequence Analytic Redundancy Relations

The ARR based hybrid diagnoser/observer uses the set of mode-specific ARRs and evaluates them concurrently (Bayoudh *et al.*, 2008b). This approach thus concurrently evaluates mode hypotheses with constant mode only. This is in contrast to hybrid observers which evaluate *mode-sequence hypotheses*. However, if one interprets a hybrid system as a *time-variant* dynamic system, one is brought to the idea to formulate ARRs for mode-sequence hypotheses as well (Domlan *et al.*, 2007).

Basis for these ARRs is a time-variant formulation of the matrices $\mathbf{O}$ and $\mathbf{L}$. Let us illustrate these matrices for our example (6) with $n_x = 2$ and an observation window defined through $p = 2$ and the associated mode-sequence $\mathcal{Q} = \langle q_\nu, q_\xi, q_\iota \rangle$, in which "$\rightarrow$" indicates the mode successor operator

$$\mathcal{Q}: \quad \mathbf{x}_{d,k-2} = q_\nu \rightarrow \mathbf{x}_{d,k-1} = q_\xi \rightarrow \mathbf{x}_{d,k} = q_\iota$$

$$\mathbf{O}(\langle q_\nu, q_\xi, q_\iota \rangle) := \left[\begin{array}{c} \mathbf{C}_\nu \\ \mathbf{C}_\xi \mathbf{A}_\xi \\ \mathbf{C}_\iota \mathbf{A}_\iota \mathbf{A}_\xi \end{array}\right] \tag{21}$$

$$\mathbf{L}([q_\nu, q_\xi, q_\iota \rangle) := \left[\begin{array}{ccc} \mathbf{D}_\nu & \mathbf{0} & \mathbf{0} \\ \mathbf{C}_\xi \mathbf{B}_\xi & \mathbf{D}_\xi & \mathbf{0} \\ \mathbf{C}_\iota \mathbf{A}_\iota \mathbf{B}_\xi & \mathbf{C}_\iota \mathbf{B}_\iota & \mathbf{D}_\iota \end{array}\right]. \tag{22}$$

In analogy to (14) we can define the residual for the mode sequence $\mathcal{Q}$ as

$$\mathbf{r}_{\mathcal{Q},k} := \mathbf{\Omega}(\mathcal{Q}) Y_k - \mathbf{\Omega}(\mathcal{Q}) \mathbf{L}(\mathcal{Q}) U_k \tag{23}$$

with the matrix

$$\mathbf{\Omega}(\mathcal{Q}) = \text{Null}\left((\mathbf{O}(\mathcal{Q}))^T\right)^T \tag{24}$$

These ARRs allow a more detailed observation of the hybrid system. However, one has to keep in mind that a hybrid system with l modes possibly implies a large number of

$$l^{p+1} \tag{25}$$

possible ARRs for the hybrid diagnosis/observation. Our simple 2-mode example already provides $2^3 = 8$ ARRs. As a consequence, it can be difficult if not even prohibitive to evaluate all ARRs concurrently. However, a carefully selected set of ARR tests for the most likely mode sequence hypotheses can provide the basis for a detailed hybrid diagnosis/estimation.

Fig 5 shows the six residuals for the mode sequences

$$\langle q_1, q_1, q_1\rangle,\ \langle q_1, q_1, q_2\rangle,\ \langle q_1, q_2, q_2\rangle,$$
$$\langle q_2, q_2, q_2\rangle,\ \langle q_2, q_2, q_1\rangle,\ \langle q_2, q_1, q_1\rangle.$$

Obviously $\mathbf{r}_{\langle 1,1,1\rangle,k}$ and $\mathbf{r}_{\langle 2,2,2\rangle,k}$ are identical to $\mathbf{r}_{1,k}$ and $\mathbf{r}_{2,k}$, respectively and provide the same information about transition occurence. But one can easily see that the mode transition $q_1 \to q_2$ can be identified through the residuals $\mathbf{r}_{\langle 1,1,2\rangle,k}$, $\mathbf{r}_{\langle 1,2,2\rangle,k}$ whereas the transition $q_2 \to q_1$ can be uniquely detected through the residuals $\mathbf{r}_{\langle 2,2,1\rangle,k}$, $\mathbf{r}_{\langle 2,1,1\rangle,k}$. The evaluation of mode-sequence ARRs thus allows us to successfully diagnose and estimate hybrid systems with non-discernible modes of operation or failure!

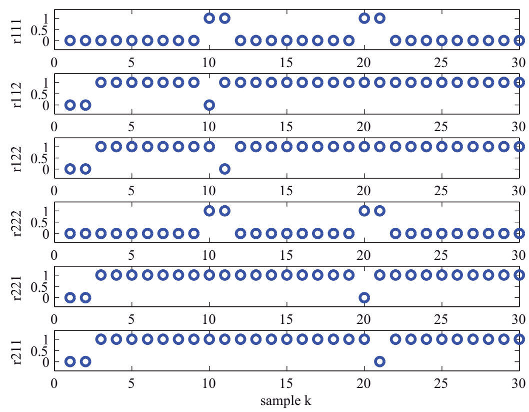

Figure 5: Residuals $\mathbf{r}_{\langle \nu,\xi,\iota\rangle,k}$ for the simulation experiment.

Now the identification is only possible if the system actually transitions. Essential for the diagnosis is hence an active behavior of the system w.r.t. its mode of operation or failure. This allows us to discriminate the modes of operation according to the specific mode-transition signature within the observation horizon. Active behavior can be achieved, for example, through *active diagnosis* which enforces mode transitions through externally triggered events so that the system uniquely reveals its operational condition.

Furthermore, capabilities of the mode-sequence ARRs suggest an extended definition of *non-discernibility*. For two modes q_i and q_j of a hybrid system (4) and an observation horizon of length $p+1$ we can define $\lambda = 2^{p+1}$ possible mode sequences $\bar{\mathcal{Q}} = \{\mathcal{Q}_0, \ldots, \mathcal{Q}_{\lambda-1}\}$. Then two modes q_i and q_j are said to be *actively non-discernible* if none of the possible mode sequences are discernible on the basis of their residuals (Domlan *et al.*, 2007).

Proposition 2 *On the basis of the mode sequence set $\bar{\mathcal{Q}} = \{\mathcal{Q}_0, \ldots, \mathcal{Q}_{\lambda-1}\}$ with mode sequences of length $p+1 = n_x+1$, two modes q_i and q_j are said to be* actively non-discernible *if for all $\mathcal{Q}_\nu \in \bar{\mathcal{Q}}$ and $\mathcal{Q}_\xi \in \bar{\mathcal{Q}}$ and the $\tilde{\Delta}(\mathcal{Q}_\nu, \mathcal{Q}_\xi)$ defined as*

$$\tilde{\Delta}(\mathcal{Q}_\nu, \mathcal{Q}_\xi) := \mathbf{L}(\mathcal{Q}_\nu) - \mathbf{L}(\mathcal{Q}_\xi) \tag{26}$$

it is true that

$$\begin{aligned} &\mathit{Rank}(\mathbf{O}(\mathcal{Q}_\nu)) = \mathit{Rank}(\mathbf{O}(\mathcal{Q}_\xi)) \\ &\quad = \mathit{Rank}\left(\left[\mathbf{O}(\mathcal{Q}_\nu), \mathbf{O}(\mathcal{Q}_\xi), \tilde{\Delta}(\mathcal{Q}_\nu, \mathcal{Q}_\xi)\right]\right). \end{aligned} \tag{27}$$

3.3 Hypotheses Filtering for hybrid Observation

Hybrid observation with hME (Hofbaur and Williams, 2004) proceeds through focussed selection of the most likely mode-sequence hypotheses and a consecutive continuous filtering process. In (Rienmüller *et al.*, 2009) we were able to show that the additional evaluation of ARRs can significantly improve the filtering capabilities of the hybrid observation algorithm. However, we only used mode-specific ARRs for this combined algorithm. The algorithm that was proposed utilizes two modes of operation, (a) an ARR improved focusing mode and (b) a bypass mode (standard hME) in the vicinity of detected mode transitions. Using the mode-sequence ARRs we can now incorporate the ARRs in the hME algorithm in a simpler and more elegant way.

Our standard hME algorithm achieves its computational efficiency through two carefully interwoven search processes. The exponential explosion of possible mode-sequences is handled through a top-level beam search process that limits the number of estimation hypotheses at each time step. Fig. 6 illustrates this operation graphically for an upper limit of $\alpha = 4$ hypotheses at each time step. Each node represents a specific estimation hypothesis with its associated state estimate. A path in the graph defines a specific mode-sequence with an associated continuously valued (discrete-time) evolution of the system's state. The hybrid estimation is done recursively and extends step-by-step the graph as the time evolves. This involves the computation of possible successor-hypotheses/-states which is done through an underlying search process. This process deduces the most likely successor-states and thus avoids the computationally expensive continuous filtering process for unlikely hypotheses as good as possible. We use an A* search procedure that consecutively deduces a successor-hypothesis on the basis of the DES Model (3) and classifies its likelihood according to the transition likelihood and the consecutive continuous filtering step. The algorithm derives the necessary continuous filter (Kalman Filter) during run-time or retrieves it from a cache to deal with systems with many modes. The evaluation of the filter provides the continuous state estimate $\hat{\mathbf{x}}_k$ sought for.

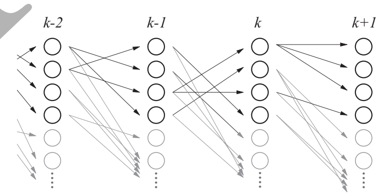

Figure 6: Beam-search process of hME.

Our proposed improvement is now to include an additional consistency test with mode-sequence ARRs *prior* to the continuous filtering process in the hME

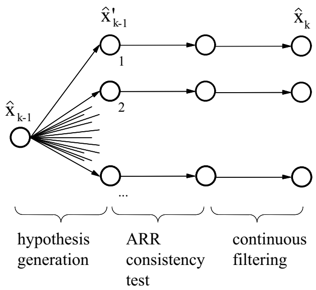

Figure 7: A* search-tree for hypothesis-extension.

search procedure as shown in Fig. 7. Of course, this means that we perform a continuous estimation step twice, ARR based and filter based! However, one obtains the additional capability to distinguish between non-discernible behaviors or behaviors with very similar input/output behavior. Even more important, we can dismiss many estimation hypotheses and thus prevent the continuous filtering process for these hypotheses early in the search process. This additional focusing capability can compensate the additional computations necessary for ARR evaluation. For (memory) efficiency purposes we also use on-line ARR deduction and caching in our algorithm to deal with complex hybrid systems.

3.4 Example

Figure 8 shows the behavior for the extended hME algorithm with the 2-mode example introduced above and the simulated hybrid trajectory of Fig. 3. We further assume that the transition event σ is not observable, thus we have to deduce the estimate from the continuous measurements only.

Figure 5 already indicated that the two modes of our system are actively discernible. We can proof this fact with the mode sequences $\mathcal{Q}_0 = \langle q_1, q_1, q_1 \rangle$ and $\mathcal{Q}_1 = \langle q_1, q_1, q_2 \rangle$ that provide distinct residuals because the associated matrices

$$\mathbf{O}(\mathcal{Q}_0) = \begin{bmatrix} 1 & 1 \\ 0.8 & 0.5 \\ 0.64 & 0.25 \end{bmatrix}, \ \mathbf{O}(\mathcal{Q}_1) = \begin{bmatrix} 1 & 1 \\ 0.8 & 0.5 \\ -0.96 & 0.9 \end{bmatrix} \tag{28}$$

span linear independent spaces which leads to

$$\text{Rank}(\mathbf{O}(\mathcal{Q}_0)) \neq \text{Rank}\left([\mathbf{O}(\mathcal{Q}_0), \mathbf{O}(\mathcal{Q}_1)]\right) \tag{29}$$

so that condition (27) does not hold.

We initiate the observer at time-step $k = 3$ with no mode information, i.e. both modes q_1 and q_2 are equally likely estimates for the time-steps $k \leq 2$. Hypothesis generation of hME extends the two initial estimates with 2 successor hypotheses for each.

$$\begin{array}{lcl} \langle q_1, q_1, q_1 \rangle & \rightarrow & \{\langle q_1, q_1, q_1 \rangle, \langle q_1, q_1, q_2 \rangle\} \\ \langle q_2, q_2, q_2 \rangle & \rightarrow & \{\langle q_2, q_2, q_2 \rangle, \langle q_2, q_2, q_1 \rangle\} \end{array} \tag{30}$$

The ARR consistency check can exclude two hypotheses ($\langle q_1, q_1, q_2 \rangle$ and $\langle q_2, q_2, q_1 \rangle$) and hME proceeds with the mode-sequence hypotheses ($\langle q_1, q_1, q_1 \rangle$ and $\langle q_2, q_2, q_2 \rangle$) and computes an associated continuous estimate $\hat{\mathbf{x}}$ for both hypotheses. hME proceeds in this

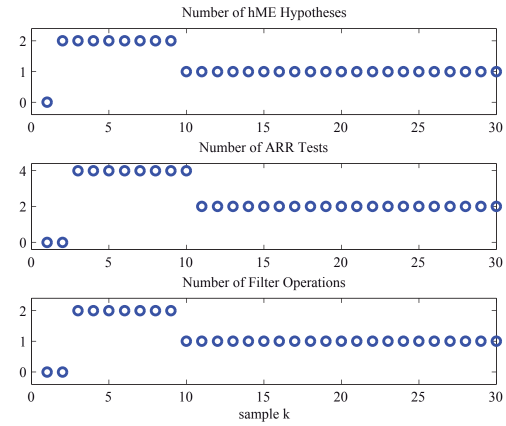

Figure 8: Simulation result with the extended hME Algorithm.

way up to the first mode change ($k = 10$). The transient continuously-valued behavior immediately after the mode transition provides enough evidence so that the ARR evaluation reduces the four hypotheses (30) to the single (correct) mode sequence hypothesis $\langle q_1, q_1, q_2 \rangle$. Once the algorithm became focused onto the correct mode of operation, it remains focused due to the interplay between DES based successor generation, ARR pre-selection and continuous filtering.

4 SUMMARY AND DISCUSSION

In model-based diagnosis, ARRs have always been used as a tool to check the consistency of observations w.r.t. a given specific model, representing the behavior of the system. This has been extended to multimode and hybrid systems for checking the consistency w.r.t. the different models. ARRs can be extended even further to fit the mode transitional behavior of hybrid systems. The proposed mode-sequence ARRs provide a deeper insight into the dynamics of the system and allow one to distinguish classically known as non-discernible modes. Some technical issues remain to be further investigated, for example what if the different models do not have the same order? Would it be more efficient for active diagnosis to consider mode-sequence ARRs involving multiple transitions? Does it impact on the temporal window to be considered? These questions, and other aspects of the mode-sequence ARRs are subject to ongoing research.

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